Phase description of limit-cycle oscillators subjected to strong perturbations

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Synchronization phenomena of nonlinear limit-cycle oscillators are ubiquitous in nature and of great interest in many fields of science and engineering [1]. To explore their synchronization dynamics, the phase reduction method, which gives approximate description of limit-cycle oscillators based only on their phase variables, has proven to be a powerful tool [2]. However, the conventional formulation is applicable only when the perturbation is sufficiently weak. In this study, we propose a generalized phase reduction method, which is also applicable to limit-cycle oscillators subjected to strong perturbation, under the assumption that the perturbations can be decomposed into a slowly varying component and remaining weak fluctuations. We show that the generalized phase reduction method can be used to analyze the dynamics of strongly perturbed oscillators that cannot be treated by the conventional method. As an example, we analyze phase locking dynamics of limit-cycle oscillators driven parametrically by strong periodic forcing, in which the orbit of the oscillator is largely deformed because of the forcing.

1. A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, 2003).
2. Y. Kuramoto, *Chemical Oscillations, Waves and Turbulence* (Dover, New York, 2003).