Phase sensitivity function for hybrid limit-cycle oscillators via the adjoint method

Sho Shirasaka [1], *Wataru Kurebayashi* [1], *Hiroya Nakao* [1]
[1]Department of Mechanical and Environmental Informatics, Tokyo Institute of Technology, Ookayama, Meguro-ku, Tokyo, Japan

Hybrid systems are dynamical systems characterized by coexistence of continuous and discrete dynamics [1]. They are utilized to describe the behavior of systems that contains some kind of discontinuous events. Such systems arise in diverse area, including impacting mechanical systems, control theory, power electronics, chemical process systems, biology and economics. Many hybrid systems exhibit periodic behavior because discontinuous events can trap the evolving continuous state within a constrained region of state space. Therefore, there emerges a new class of stable limit-cycling behavior which is induced by discontinuity, and whose issue of synchrony remains to be addressed. Phase-reduction method provides a useful framework for the analysis of stable limit-cycle oscillators [2,3]. As long as the perturbation is sufficiently weak, the dynamics of the limit-cycles can be captured quantitatively by a scalar phase equation. The phase reduction method has been widely used to study synchronization properties of various types of oscillators [4]. However, the adjoint method [5], which provides numerically accurate linear phase response of oscillator ' s phase to applied perturbation, called phase sensitivity function, cannot be applied to hybrid limit-cycle oscillators straightforwardly due to the non-smoothness of the vector field of hybrid systems. In this study, we develop an extension of the conventional adjoint method to hybrid systems, which enables us to find semi-analytically a phase sensitivity function from a set of equations of the system.

1. M. Bernardo and C. Budd and A. R. Champneys and P. Kowalczyk, *Piecewise-smooth dynamical systems: theory and applications* (Springer, Berlin, 2007).
2. A. T. Winfree, *The geometry of biological time* (Springer, Berlin, 2001).
3. Y. Kuramoto, *Chemical Oscillations, waves, and turbulence* (Dover, New York, 2003).
4. A. Pikovsky and M. Rosenblum and J. Kurths, *Synchronization: a universal concept in nonlinear sciences* (Cambridge university press, Cambridge, 2003).
5. I. G. Malkin, *Methods of Poincare and Liapunov in theory of non-linear oscillations* (Gostexizdat, Moscow, 1949).